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Nonlinear Flutter Analysis of Wings at High Angle of Attack

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Introduction

TO consider the nonlinear flutter characteristics of wings at high angle of attack, Strganac and Mook[1] developed an integration method in the time domain. With the unsteady airloads on wings obtained by an unsteady vortex-lattice method, the equation of motion for the moving wing is integrated in the time domain. In Ref. 1, only the results of a rectangular wing with large aspect ratio (AR = 10) were given. For another low aspect ratio rectangular wing, no detailed results were supplied. The integration method in Ref. 1 can give the vibration history of wings at any flying speeds, but this computation procedure is time consuming.

By introducing a describing function for the nonlinear generalized aerodynamic force, Ueda and Dowell[2] analyzed the flutter of airfoils at transonic flow. This method is based on the frequency domain, and has better computational efficiency.

Presently, the phenomenon of nonlinear flutter for wings with separated vortex at high angle of attack has not been thoroughly investigated. Therefore, wind-tunnel tests for this problem would be helpful for further research.

In this Note, both time integration method and describing function method are developed for the nonlinear flutter analysis of wings at high angle of attack. The needed nonlinear airload distribution is calculated by a subsonic unsteady method—potential difference method,[3] developed recently by the authors. Additionally, the experiments of flutter for the wings at high angles of attack are conducted, and the test results confirm the feasibility of the methods mentioned above.

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Time Integration Method (TIM)

The small structural deformation of a wing can be expressed by

$$Z(x, y, t) = \sum_{i=1}^{N} \Phi_i(x, y)\cdot q_i(t) \tag{1}$$

where $\Phi_i(x, y)$ is the ith eigenmode, and $q_i(t)$ is the corresponding generalized coordinate.

Then, the equation of motion can be written as

$$(M)\cdot(\ddot{q}) + (K)\cdot(q) = (f) \tag{2}$$

where (M) and (K) are the generalized mass matrix and stiffness matrix, respectively, (f) is the generalized aerodynamic force vector.

By introducing the state vector $(e) = (q_1, q_2, \ldots, q_N, \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_N)^T$, Eq. (2) is transformed to

$$(\dot{e}) = \begin{bmatrix} (O) & (I) \\ (M)^{-1}(K) & (O) \end{bmatrix}\cdot(e) + \begin{bmatrix} (O) \\ (M)^{-1} \end{bmatrix}\cdot(f) \tag{3}$$

For a certain vector (e), the aerodynamic pressure difference $\Delta P(x, y)$ on the wing surface can be computed by the potential difference method, and the generalized aerodynamic forces are calculated by

$$f_i = \int\int_S \Delta P(x, y)\cdot\Phi_i(x, y)\cdot dx\ dy \tag{4}$$

Equation (3) can be solved by the well-known Runge-Kutta method. The state vector (e) obtained at the end of each time step can provide a new initial condition for the next time step to calculate the needed aerodynamic pressure distribution. By this procedure, the history of the wing motion can be simulated in the time domain.

It should be noticed that for a given basic angle of attack, different from the linear problem, the static deformation (q_s) must be taken into account at first by an iteration process. The governing equation is

$$(K)\cdot(q_s) = (f_s) \tag{5}$$

where (f_s) is the static aeroelastic airload. Then, the initial state vector (e_0) for the above Runge-Kutta procedure is formed with (q_s) superposed by a certain disturbance.

Describing Function Method (DFM)

For a harmonic vibration of a wing $\xi = \bar{\xi}e^{ik\tau}$, where τ is the nondimensional time and k is the reduced frequency, the corresponding generalized aerodynamic force f is expressed by

$$f = \tfrac{1}{2}\rho_\infty U_\infty^2\cdot S\cdot C(\xi, \tau) \tag{6}$$

where $\frac{1}{2}\rho_\infty U_\infty^2$ is the dynamic pressure, S is the wing area, and $C(\xi, \tau)$ is the coefficient of generalized aerodynamic force.

For present problem, $C(\xi, \tau)$ is a nonlinear function with respect to ξ. For a harmonic motion, it can be linearized by introducing a describing function $D(\bar{\xi}, ik)$, then, the following approximate relation exists:

$$C(\xi, \tau) = D(\bar{\xi}, ik)\cdot\xi = D(\bar{\xi}, ik)\cdot\bar{\xi}e^{ik\tau} \tag{7}$$

Table 1 Comparison of calculated results with experiments

Case no.	Test wing	α_0	ω_ψ, Hz	ω_θ, Hz	V_F, m/s Test	TIM	DFM
1	Rectangular	14	1.875	2.5	11	10.25	10.47
2	Rectangular	18	1.875	2.5	10	9.25	9.129
3	Rectangular	14	1.875	2.75	13	11.30	11.9635
4	Rectangular	18	1.875	2.75	11	10.20	10.1936
5	Delta	14	2.0	2.375	21	19.50	18.6414
6	Delta	18	2.0	2.375	20	18.50	17.56

The corresponding relation in the Laplace domain is

$$\hat{C}(\bar{\xi}, s) = \frac{D(\bar{\xi}, s)\cdot\bar{\xi}}{s - ik} \tag{8}$$

If $k = 0$, Eq. (8) represents an indicial response relationship

$$H(\bar{\xi}, s) = \frac{D(\bar{\xi}, s)\cdot\bar{\xi}}{s} \tag{9}$$

In practical application, the indicial response of aerodynamic force in time domain is first calculated by giving a certain value $\bar{\xi}$. Then, with curve fitting, this indicial response is approximated by a polynomial of exponential functions

$$\sum_{n}^{M} a_n \cdot \exp(b_n \tau)$$

from which a closed form expression of $H(\bar{\xi}, s)$ can be obtained. According to Eq. (9), the describing function is

$$D(\bar{\xi}, ik) = \frac{ik \cdot H(\bar{\xi}, ik)}{\bar{\xi}} \tag{10}$$

By introducing the describing functions, the generalized aerodynamic force vector for wings at high angle of attack formally has a linear relation to the generalized displacements

$$(f) = \tfrac{1}{2}\rho_\infty U_\infty^2 \cdot S \cdot (D) \cdot (q) \tag{11}$$

where the generalized aerodynamic force coefficient matrix (D), different from the linear problem, depends on not only the reduced frequency, but also the vibration amplitudes.

From Eqs. (2) and (11), a linear flutter equation can be obtained

$$(M)\cdot(\ddot{q}) + (K)\cdot(q) = \tfrac{1}{2}\rho_\infty U_\infty^2 \cdot (D) \cdot (q) \tag{12}$$

This equation must be solved by an iteration process. At first, some vibration amplitudes $(\bar{q}_0)$ are chosen. After the steady flow for the wing at a certain basic angle of attack is computed, a step deformation $\Phi_j(x, y)\cdot\bar{q}_{0j}$ is superposed upon the wing impulsively, and the indicial responses of generalized aerodynamic force coefficients for different modes are obtained in the time domain. Therefore, the D_{ij} $(i = 1, N)$ are calculated by the procedure mentioned above.

Having gotten the aerodynamic matrix (D), the conventional V-g method is adopted to solve Eq. (12), and the results can provide a critical flutter speed V_F and the ratio of the mode amplitudes $(\bar{q}_F)$, which usually is not consistent with the preassigned $(\bar{q}_0)$. Then, the $(\bar{q}_F)$ is used as the new initial $(\bar{q}_1)$ for the second time of the computation, and this procedure will proceed until the convergent results are obtained.

Wind-Tunnel Experiments

The models for the wind-tunnel test are a solid wooden rectangular wing and a delta wing. A supporting system is designed to provide the rolling ψ and pitching θ DOF for the rigid models. The rectangular wing has a sharp side edge with aspect ratio AR = 2, and the chord length is 300 mm. The pitching axis is 68 mm behind the leading edge. The delta wing has a sharp leading edge with aspect ratio AR = 2.61, and the root chord length is 460 mm. The pitching axis is 285 mm ahead the trailing edge. The basic angles of attack in the experiments are 14 and 18 deg for both wings. In this range of angle of attack, separated vortex occurs from the side edge for the rectangular wing and from the leading edge for the delta wing.

A low-speed wind tunnel is used. The diameter of the test section is 1 m, and the maximum airspeed of the tunnel is 60 m/s. The supporting springs are adjustable, which results in different eigenfrequencies for the model. The ω_ψ and ω_θ denote the rolling and pitching frequency, respectively. For the rectangular wing, two sets of ω_ψ and ω_θ are selected. One set is ω_ψ = 1.875 Hz and ω_θ = 2.5 Hz. Another set is ω_ψ = 1.875 Hz and ω_θ = 2.75 Hz. For the delta wing, only one set (ω_ψ = 2.0 Hz and ω_θ = 2.375 Hz) is used.

Results and Conclusions

The test results of the critical flutter speed and the results calculated by both TIM and DFM are listed in Table 1, where α_0 is the basic angle of attack.

For the calculation of the generalized aerodynamic force coefficients in the DFM, a polynomial with 11 terms of exponential functions is used in the curve fitting process.

Compared with the TIM, the advantage of the DFM lies in the much less computational time needed to get a flutter point. But the DFM can only give the critical flutter point, while the TIM can give not only the critical flutter point, but also the subcritical and supercritical responses. Besides, the linearization approximation of the generalized aerodynamic forces in the DFM is not involved in the TIM. The superposition of the modal generalized aerodynamic forces in the DFM is tenable only upon the engineering consideration.

From the results of the present work, it is shown that the higher the basic angle of attack of the wing, the lower the critical flutter speed. This means that the separated vortex for the wings at high angle of attack deteriorates the flutter characteristics of the wing, which is important for combat aircrafts.

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